

These results agree with the data taken by Aroesty and Berger from their Ref. 15 and exhibited in their Fig. 3. Thus, rather than a ΔT of 40°F for a β of -0.24 indicated by Aroesty and Berger, it is in fact a ΔT of about 140°F. Predictably, the errors just listed result in a totally incorrect result.

References

¹Aroesty, J. and Berger, S. A., "Controlling the Separation of Laminar Boundary Layers in Water: Heating and Suction," *Journal of Hydraulics*, Vol. 11, No. 3, July 1977, pp. 107-111.

²Wortman, A. and Mills, A. F., "Separating Self-Similar Laminar Boundary Layers," *AIAA Journal*, Vol. 9, Dec. 1971, pp. 2449-2451.

Reply by Authors to A. Wortman

J. Aroesty*

The Rand Corporation, Santa Monica, Calif.

and

S. A. Berger†

University of California, Berkeley, Calif.

THE intent of our paper was to explore the potential impact of surface heating on the separation of the

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*Senior Research Engineer, Physical Sciences Department.

†Professor, Mechanical Engineering Department; also consultant to The Rand Corporation. Member AIAA.

laminar boundary layer in water. Since our approach was analytic, it was necessary to simplify the appropriate equations of motion while still retaining the dominant phenomena. It is well known that for water the temperature variation of viscosity is much greater than the corresponding variations of thermal conductivity. (For example, kinematic viscosity increases by 77% between 68°F and 32°F, while the thermal diffusivity decreases by only 8.5%). Thus, the essential phenomena are due to viscosity variation and large Prandtl number, but not the temperature dependence of thermal conductivity. For purposes of developing simple estimates and closed form solutions, it is then appropriate to approximate the thermal conductivity by a constant value, but to retain the temperature variation of the viscosity. Contrary to Dr. Wortman's assertion, this is exactly the procedure described in our paper.

The boundary layer energy equation can then be simplified to

$$g'' + Pr_{\infty} f g' = 0$$

but the viscosity variation with temperature has in fact been included, using the form $1/N = (a + bT) / (a + bT_{\infty})$. (See our Eq. 11).

Our asymptotic analysis, valid for large Prandtl number, overestimates the impact of heating on separation for the case of water where the magnitude of $(Pr)^{1/3}$ is only two. This was recognized at the time of publication, and therefore we included a revised relationship between surface overheat and pressure gradient parameter, shown in our Fig. 3. This relationship is essentially the one which Wortman recommends. We do not challenge him on this. We merely point out that it was already included in our paper.

The conclusion that heating has little potential for maintaining attached laminar flow in water was based on our analytic approach. The later availability of numerical solutions served to strengthen this conclusion even further.